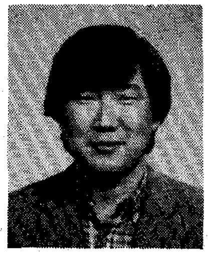
Han S. Uhm received the B.S. degree from Seoul National University, Seoul, Korea, in 1969, the M.S. degree from the University of Maryland, College Park, in 1973, and the Ph.D. degree also from the University of Maryland in 1976, all in physics.

Since joining the Naval Surface Weapons Center in 1978 in the Nuclear Branch, he has worked as a Plasma Research Physicist. His research has included work on charge beam propagation in a background plasma, collective ion acceleration, beam properties in a particle accelerator, high-power microwave generation, non-neutral plasma, thermonuclear magnetic confinement, and electron and positron beams in a storage ring.



Joon Y. Choe was born in Seoul, Korea, and received the B.S. and Ph.D. degrees in physics from the Seoul National University and the University of Kansas, Lawrence, respectively.

He worked on fusion plasma stability at the Courant Institute and the University of Maryland, College Park, and on microwave generation at the Naval Research Laboratory. Since joining the Naval Surface Weapons Center, he has been engaged in research on microwave generation, waveguide theory and experiment, particle accelerator theory, and computer simulations.

Mode-Specific Reflectometry in a Multimode Waveguide

DAVID S. STONE, MEMBER, IEEE, KEVIN L. FELCH, AND STEPHEN T. SPANG, MEMBER, IEEE

***Abstract*—A technique for measuring the voltage-standing-wave ratio (VSWR) created by a mismatch for a specific mode in a multimode waveguide is described. A heavily loaded resonant cavity is used to launch the mode of interest and the variation in the cavity loaded Q is noted as the phase separation of the cavity and the mismatch is varied. The bandwidth of this technique is generally about 0.03 percent and VSWR as low as 1.05:1 may be measured accurately. Mode-specific VSWR measurements are of particular interest in analyzing the performance of multimode waveguide components, and in optimizing multimode networks. The measurement technique may be used, for example, in the design and optimization of transmission lines for electron cyclotron resonance heating systems in magnetic fusion devices.**

I. Introduction

RECENT ADVANCES in high average power millimeter-wave devices, such as the gyrotron [1], [2], have aroused interest in the use of multimode waveguides for handling power levels which would produce excessive power densities in single-mode systems. Millimeter-wave sources of this type are susceptible to mode competition [3], which is aggravated by reflections of either the principal mode of interest or of the competing modes. It is, therefore, particularly important to be able to characterize a multimode waveguide system by measuring the voltage-standing-wave ratio (VSWR) for several specific modes of interest in the system.

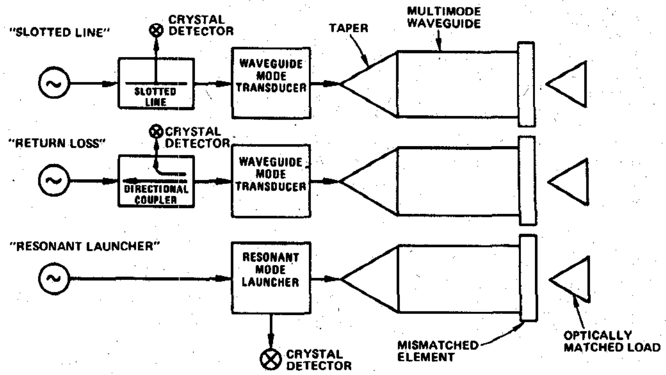


Fig. 1. Schematic diagram of three VSWR measurement techniques in multimode waveguide.

Conventional reflection measurements in waveguides are made using commercially available transducers to launch the waveguide mode of interest (see Fig. 1). Standard millimeter-wave test equipment is designed for the $TE_{10}^{\square}$ (fundamental rectangular) mode, whereas overmoded transmission lines, as employed in long-distance telephone links [4], or gyrotron collector structures [5], are typically cylindrical. Thus transducers designed to produce mode transformations of the type $TE_{10}^{\square} \leftrightarrow TE_{mn}^{\circ}$ and $TE_{10}^{\square} \leftrightarrow TM_{mn}^{\circ}$ are required. Only the simplest of these waveguide components, the $TE_{10}^{\square} \leftrightarrow TE_{11}^{\circ}$ transducer is available with

Manuscript received September 15, 1982; revised April 15, 1983. This work was supported by a grant from the Oak Ridge National Laboratory, which is operated by the Union Carbide Corporation for the U.S. Department of Energy under Contract W-7405-eng-26.

The authors are with Varian Associates, Inc., 611 Hansen Way, Palo Alto, CA 94303.

0018-9480/83/0900-0710$01.00

good broad-band performance (~ 30-percent bandwidth). The transformation $TE_{10}^{\square} \leftrightarrow TE_{01}^{\circ}$ can be achieved with Marie-type transducers [6] (min VSWR 1.2 across 20-percent bandwidth) or slot-coupled transducers (min VSWR 1.1 across 2-percent bandwidth). Higher order modes may be launched with phase velocity couplers of the type used by Miller [7] and more recently by Moeller [8], but these are expensive and cumbersome to use. Zone plate transducers may also be employed [9], but at the quasi-optical frequencies of interest here, these launch unacceptable levels of spurious modes. Shimada [10] has described a high-Q cavity transducer which, while mechanically tunable over a 10-percent bandwidth, suffers from high transfer loss (1–3 dB) and input VSWR (up to 1.8:1) and, therefore, is only suitable for measurement of large VSWR's (> 3:1). Thus no suitable means of launching pure higher order circular electric modes is available for use with standard test equipment. This circumstance has led us to the development of a technique for measuring the VSWR of a multimode waveguide mismatch for any particular higher order mode.

A heavily loaded resonant cavity is used to launch the mode of interest into the waveguide system. The variation in the loaded Q of the cavity is then noted as the phase separation of the mismatch and the cavity is changed. It is shown that the mode-specific VSWR may be determined directly from the ratio of the maximum and minimum values of the signal received at the detector port of the launcher. The technique is the multimode waveguide analog of the standing-wave indicator technique commonly used with single-mode waveguide systems. A pick-up stub in a slotted line senses the standing-wave pattern in the latter case, while the multimode cavity acts as the probing element in the former case. Movement of the probing element along the guide through the standing-wave maxima and minima is accomplished in the conventional measurements by physically moving the pick-up stub in the slotted line. Similarly, in the present technique, the resonant mode launcher may be moved physically to observe the standing-wave ratio. In addition, if the physical separation of the cavity and the mismatch is large enough, it is equivalent to electronically vary the phase separation of the cavity and the mismatch.

In the following section, we analyze the behavior of the resonant cavity with a mismatch in the output waveguide, using an equivalent circuit analysis. In Section III we elaborate on the details and limitations of applying this VSWR measurement procedure for both short and long multimode transmission lines. Finally, in Section IV we measure the mode-specific VSWR for various mismatches.

II. Circuit Analysis

The mode-specific VSWR measurement technique consists of varying the phase separation of the launcher and the mismatch through π radians or until Z_L the load impedance appears entirely resistive at the right-hand end of the cavity. This phase change may be accomplished by physically varying the distance L separating the cavity launcher and the mismatch, or by varying the generator frequency as we will discuss in more detail in the next section.

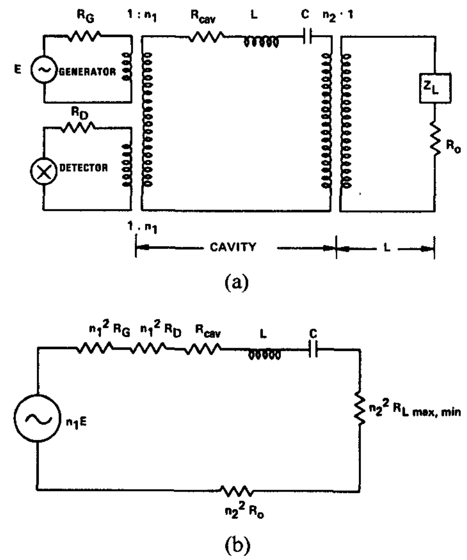


Fig. 2. (a) Equivalent circuit of cavity launcher coupled to an unknown mismatch. (b) Equivalent circuit transferred into the cavity.

The equivalent circuit for the cavity launcher and unknown load mismatch is shown in Fig. 2(a). A source with impedance R_G is coupled to the cavity through a transformer with a turns ratio of $1:n_1$.

A detector with an impedance R_D is also coupled to the cavity using the same transformer. The resonant circuit of the cavity is represented by R_{cav}, L, and C. The unknown mismatch, Z_L and the matched load R_o are coupled to the output of the cavity through a second transformer with a turns ratio of $n_2:1$.

The two purely resistive load impedances found by varying the phase through π radians, which we call $R_{L\max}$ and $R_{L\min}$, may be related to the VSWR through the following relationship:

$$\frac{R_{L\max}}{R_o} = \frac{R_o}{R_{L\min}} = \text{VSWR}. \tag{1}$$

In Fig. 2(b) we show all of the loads as seen transformed into the cavity. The loaded Q of the cavity, Q_L is now easily calculated

$$\begin{aligned} Q_L &= \frac{\omega_o L}{R_{\text{total}}} \\ &= \frac{\omega_o L}{n_1^2 R_G + n_1^2 R_D + R_{\text{cav}} + n_2^2 R_L + n_2^2 R_o} \end{aligned} \tag{2}$$

where $\omega_o = 1/\sqrt{LC}$.

We now make the assumption that the cavity is very lightly coupled to the generator and detector ($n_1^2 \ll 1$) and that the loaded Q is much less than the ohmic Q of the cavity (denoted Q_o), so that

$$\begin{aligned} Q_o &\gg Q_L \\ R_o &\gg R_{\text{cav}}. \end{aligned} \tag{3}$$

With these assumptions, (2) may be written

$$Q_L = \frac{\omega_o l}{n_2^2(1+r_L)} \tag{4}$$

where

$$l \equiv \frac{L}{R_o} \text{ and } r_L \equiv \frac{R_L}{R_o}.$$

The ratio of the maximum loaded Q to the minimum loaded Q is given by

$$\frac{Q_{L\max}}{Q_{L\min}} = \frac{1+r_{L\max}}{1+r_{L\min}}. \tag{5}$$

Using (1), we see that

$$\frac{Q_{L\max}}{Q_{L\min}} = r_{L\max} = \text{VSWR}. \tag{6}$$

The power as seen by the detector P_D may be written

$$P_D = n_1^2 R_D I^2 \tag{7}$$

where the current, I, may be calculated by looking at Fig. 2

$$I = \frac{n_1 E}{n_1^2 R_G + n_1^2 R_D + R_{\text{cav}} + n_2^2 R_L + n_2^2 R_o + j\left(\omega L - \frac{1}{\omega C}\right)}. \tag{8}$$

Here, E is the RF electric field of the generator signal. Now, assuming that we are on a resonant frequency of the cavity, $\omega = \omega_o = 1/\sqrt{LC}$, and using (2), we may solve for I in terms of Q_L

$$I = \frac{Q_L n_1 E}{\omega_o L}. \tag{9}$$

Finally, if we combine (6), (7), and (9), we obtain

$$\frac{P_{D\max}}{P_{D\min}} = \frac{|I_{\max}|^2}{|I_{\min}|^2} = \left(\frac{Q_{L\max}}{Q_{L\min}}\right)^2 = [\text{VSWR}]^2. \tag{10}$$

This expression indicates that the ratio of the maximum and minimum power levels measured by the detector, as we vary the phase separation of the cavity launcher and the unknown load, yields a direct measurement of the VSWR of the mismatch.

III. VSWR Measurement Procedure

A. Apparatus

The experimental equipment employed in making mode-specific VSWR measurements in a multimode waveguide consists of a resonant cavity launcher, a gradual taper connecting the launcher to the multimode waveguide, and the unknown mismatch placed at the end of the multimode waveguide. A schematic diagram of the experimental setup is shown in Fig. 3.

The cavity launcher is a low Q ($\approx$100–2000) open resonant cavity similar to those used in gyrotrons [11]. For example, to measure the VSWR of an unknown mismatch at 60 GHz for the TE_{02} circular waveguide mode, a cavity launcher resonant in the TE_{021} cavity mode at 60 GHz is employed. In Fig. 4 we show the resonance characteristics of such a cavity launcher. The launcher is excited by means of a $TE_{10}^{\square}$ waveguide feeding one of several small coupling holes in the walls of the cavity equispaced about the axis of symmetry. A crystal detector is coupled to a separate, but identical, coupling hole in the same manner as the signal source, to measure the transmission resonance signal.

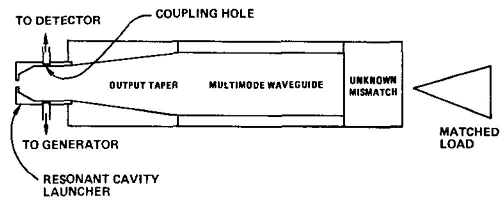


Fig. 3. Schematic diagram of the experimental setup used for measuring the mode-specific VSWR in multimode waveguides.

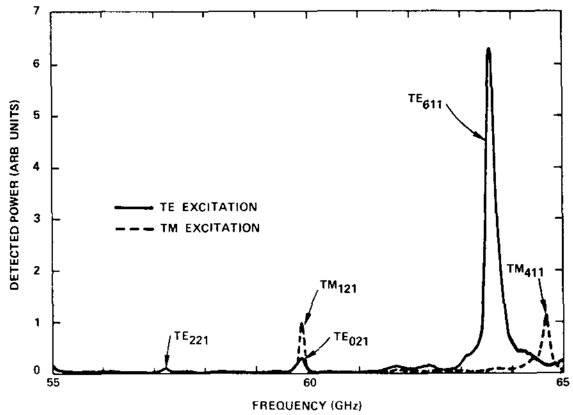


Fig. 4. Frequency response of a resonant cavity launcher for TE and TM excitation.

A gradual taper (1–10°) couples the power out of the cavity in such a way that mode conversion is avoided in connecting the launcher to the multimode waveguide. Output mode purity has been checked for these cavities using a technique described by Stone [5]. This method of coupling power to the unknown mismatch mounted in the multimode waveguide heavily loads the cavity launcher, easily satisfying our assumption that the ohmic (unloaded) Q is much greater than the loaded Q, as required by (3) in the previous section.

To perform the measurement, two methods may be employed to vary the phase separation of the mismatch and the cavity launcher. If the multimode transmission line is "short" the physical length of the line must be varied. If the line is "long" the phase separation of the cavity and the mismatch may be changed by varying the physical length of the line or by varying the generator frequency.

B. Short Line Technique ($L < cQ_L/f$)

If a "short" multimode line is used, for which $L < cQ_L/f$, where c is the speed of light in vacuum and f is the signal frequency, the VSWR must be measured by physically varying the separation L of the mismatch and the launcher. Since the round-trip phase separation is $2kL$,

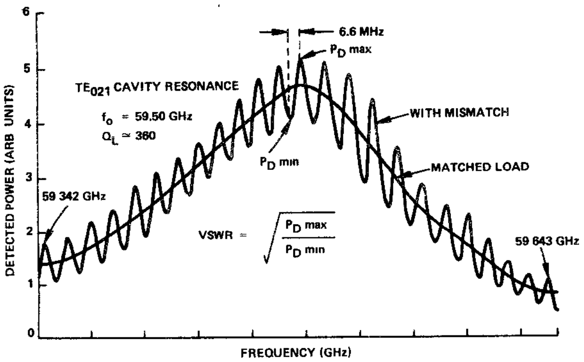


Fig. 5. Detected cavity resonance signal for a mismatch and a matched load separated from the cavity launcher by a "long" line.

where k is the wavenumber of the signal in the multimode guide, a phase variation of π rad, may be achieved by varying L through a distance $\Delta L = \lambda/4$. When a square-law detector is used for observing the launcher response, the VSWR of the mismatch may be determined by applying (10). Alternatively, if the detector output is not a known function of received power, we may use

$$\text{VSWR} = \text{antilog}\left[0.05\{P_0^{\max}(\text{dB}) - P_0^{\min}(\text{dB})\}\right]. \quad (11)$$

C. Long Line Technique ($L \gg cQ_L/f$)

For a "long" line, with $L \gg cQ_L/f$, we can employ the short line technique or we can change the optical path length sufficiently to observe the maxima and minima in launcher response by sweeping the generator frequency over a bandwidth $\Delta f \simeq c/4L$. To satisfy the long line criterion we require that $\Delta f \ll Q_L/f$. An example of one such measurement is shown in Fig. 5. In this case, the VSWR of the mismatch is again given by (10) (a square-law detector was employed here), but now we simply use the minimum and maximum signals occurring at the resonance peak. The frequency difference Δf was measured to be 6.6 MHz, in good agreement with the predicted value of $\Delta f = c/4L = 7.0$ MHz for the length of line L used in the measurement. Had a shorter line been employed, we would have been forced to use minimum and maximum signals occurring off the resonance peak, thus invalidating our assumption made in Section II that $\omega = \omega_o$. Conversely, a longer line separating the cavity launcher and the mismatch would have allowed measurement even closer to the resonance peak. The numerical error encountered in taking measurements off of the resonance peak will be discussed in the next section.

D. Limitations and Assumptions

One must consider, however, various experimental limitations on the accuracy of a measurement using the "short" or the "long" line techniques. In this section we discuss the ramifications of the simplifying assumptions made in our analysis in Section II, and the limitations specific to either the short or long line method.

1) Launched-Mode Purity: In developing our analysis of the mode-specific VSWR measurement technique, we have assumed that the launched signal consists of a single predetermined waveguide mode and that the reflected signal reaching the cavity launcher also consists of the same pure mode. Alteration of the mode of the reflected signal involves the question of mode conversion which we will address in the next section. There are two important factors that affect the purity of the launched mode: the orientation of the feed waveguide, and discontinuities in the cavity.

Careful orientation of the rectangular $TE_{10}^{\square}$ generator feed with respect to the launcher axis will ensure excitation of the desired transverse orientation. That is, for transverse electric (magnetic) modes, the electric field of the feed waveguide is aligned perpendicular (parallel) to the launcher axis. Of course, the waveguide probe coupling the detector crystal to the cavity launcher must be oriented in a like manner. Since several transverse electric modes are degenerate with certain transverse magnetic modes (i.e., TE_{0n1}° and TM_{1n1}°), special care must be taken in these cases to ensure the correct transverse excitation.

Examples of discontinuities which might affect the launched signal include machining defects in the cavity, misalignment of the cavity launcher with the output taper, and improper choice of the size and number of the coupling holes in the cavity launcher. Any such defect can impose an unwanted asymmetry in the cavity thus exciting spurious modes. Machining and alignment errors become more and more critical for higher frequencies and higher order modes. We have studied the effect of the coupling hole diameter on cavity properties and have concluded that hole diameters of the order of or less than $\lambda/4$ may be tolerated.

The number N of coupling holes, which are equispaced about the axis of symmetry, is also critical [11]. The value of N must be chosen to impose an N-fold symmetry which does not allow coupling of the cavity fields to any propagating waveguide modes. For example, when designing a launcher for the TE_{onl} cavity mode, we require $N = 3n$ because the TE_{3n1} waveguide mode is cut off for all n.

2) Cavity Coupling: We have required that the generator and detector be lightly coupled to the cavity launcher ($n_1^2 \ll 1$ in Fig. 2). This assumption must be verified experimentally by varying the size of the coupling holes until the point where the loaded Q, Q_L, of the cavity launcher, remains unaffected by the presence of the holes. As mentioned above in discussing the purity of the launched mode, a hole diameter, $d \lesssim \lambda/4$, has been found to be acceptable. We have also described the proper choice of the number N of coupling holes.

Finally, we have required the ohmic Q to be much greater than the loaded Q as stated in (3). For copper cavities, the ohmic Q, Q_o, is generally in the range, $10{,}000 \leqslant Q_o \leqslant 20{,}000$, whereas the loaded Q, Q_L, of the cavity launcher is normally in the range, $100 \leqslant Q_L \leqslant 2000$ [11]. When the mismatch is added to the circuit, one must ensure that $Q_{L\max}$ is still much less than Q_o. For example, if $Q_L = 500$ for a matched load, a severe mismatch with VSWR $= 10$ would yield a value of $Q_{L\max} = 909$, still far less than Q_o. Thus the assumption $Q_L \ll Q_o$ is satisfied under most circumstances.

3) Mode Conversion of the Reflected Wave: The topic of mode conversion in multimode waveguide has been discussed in several references [5], [12], [13]. Here, we are interested in the conversion into modes which upon reflection would enter the cavity launcher, thereby changing the observed cavity Q and introducing error into the mode-specific VSWR measurement. Morgan has shown [13] that any waveguide discontinuity of characteristic axial length l will partially convert an incident TE_{mn} wave into forward and backscattered $TE_{m'n'}$ waves. The converted wave electric field amplitudes $E_{m'n'}$ are proportional to the spatial Fourier transform of the waveguide discontinuity. That is, in a cylindrical waveguide with a radial discontinuity described by the function $d(\phi, z)$ we have

$$E^{\pm}_{m'n'} \propto \int\int d(\phi, z) e^{-iK^{\pm}(m,n,m'n')z} e^{-im\phi}\, dz\, d\phi. \quad (12)$$

Here $K^{\pm}$ is the beat wavenumber for the forward $(+)$ and backscattered $(-)$ waves beating with the incident wave. The beat wavenumbers are

$$K^{\pm}(m, n, m', n') = k_{mn} \mp k_{m'n'}. \quad (13)$$

The exponential terms in the integrand in (12) extract the characteristic axial length and azimuthal periodicity of the waveguide discontinuity from $d(\phi, z)$ in terms of beat wavenumbers and azimuthal mode numbers.

In the case where $l \gg \lambda_o$, using (12) it is easy to show that appreciable mode conversion occurs only into those forward modes which satisfy $l \sim (k_{mn} - k_{m'n'})^{-1}$. The beat wavelength of the backscattered wave with the incident wave $2\pi/(k_{mn} + k_{m'n'})$ is so short compared to l that energy cannot be coupled effectively into the backward wave. We conclude that for mismatches in multimode waveguide with discontinuities of length $l \gg \lambda_o$, including such waveguide components as waterloads, miter bends and gradual tapers, that the backscattered converted wave does not affect the accuracy of the VSWR measurements.

Now consider the case of a "short" discontinuity in multimode waveguide, with $l \lesssim \lambda_o$. The Fourier transform (12) now selects wavenumbers for the converted modes which satisfy $2\pi/(k_{mn} \mp k_{m'n'}) \approx \lambda_o$. This criterion is satisfied by modes near cutoff with $k_{m'n'} \approx 0$. Therefore, for mismatches with $l \lesssim \lambda_o$, such as mode filters and waveguide gaps, the backscattered converted wave is not necessarily of negligible amplitude, but is close to cutoff in the multimode waveguide. However, as this reflected wave encounters the down-taper which connects the cavity launcher to the multimode waveguide, it is cut off and reflected entirely. Thus the launcher does not "see" the backscattered converted wave and again only the reflected incident wave will be measured in the cavity launcher. Consequently the mode-specific VSWR measurement technique is not affected by mode conversion due to "short" discontinuities.

4) "Short" Line: Effect of Waveguide Gap: Recalling that the "short" line VSWR measurement procedure required

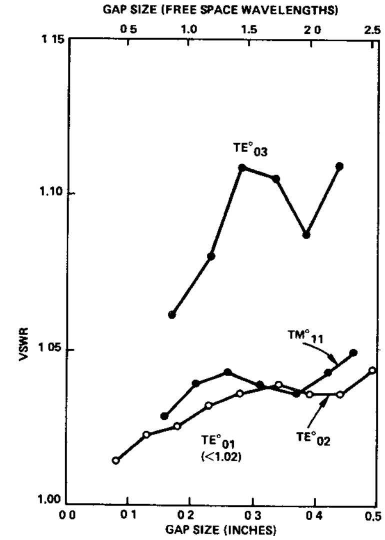


Fig. 6. Mode-specific VSWR for various modes at 60 GHz for a narrow gap in 2.5-in ID circular waveguide.

an axial displacement of the mismatch with respect to the cavity launcher in order to vary the phase of the reflected signal, we must concern ourselves with the specific effects of the small gap ($\sim \lambda/4$) created in the multimode waveguide with such a displacement. In light of the arguments in the preceding section, it is obvious that converted modes backscattered from the gap are near cutoff in the multimode waveguide and will not be detected at the cavity launcher. Calculations by Morita [12] for circular electric modes (TE_{on}) indicate that the VSWR of the small gap itself is negligible. However, to avoid erroneous measurements, the VSWR of the gap should be measured before observing the VSWR of the mismatch in question. For a gap VSWR $= 1 + \epsilon$, with $\epsilon \ll 1$, the relative error introduced in the measurement of the load VSWR will be approximately equal to ϵ.

We have measured the mode-specific VSWR of a narrow circumferential gap in 2.5-in ID circular waveguide for various different modes at 60 GHz. The results of these measurements are shown in Fig. 6. We note that, as expected, higher order modes (possessing higher wall currents) are more affected by a gap than are lower order modes. For a VSWR measurement of an unknown mismatch using the short line technique, we are interested only in gaps with $l < \lambda/2$ so that for all of the modes except the TE_{03} mode, measurements on mismatches with VSWR $\gtrsim$ 1.05 should be quite accurate. For the TE_{03} mode, more care must be taken in measuring mismatches with a VSWR below 1.10.

5) "Long" Line: Off-Resonance Correction: In the equivalent circuit analysis in Section II, we made the assumption that all measurements were performed at the resonant frequency of the cavity launcher $\omega_o = 1/\sqrt{LC}$. However, for the "long" line technique where the phase is varied by sweeping the frequency, it is necessary to stray somewhat from the resonant frequency, depending on the length of the line, to locate the minimum and maximum Q_L in the

TABLE I
EFFECT OF PERFORMING VSWR MEASUREMENTS FOR $\omega \neq \omega_0$
$f_0 = f_{max} = 59.500$ GHz

f_{min} (GHz)	$\Delta f = f_{max} - f_{min}$ (MHz)	$\left(\frac{P_{Dmax}}{P_{Dmin}}\right)^{1/2}$	% Difference from VSWR = 1.5
59.430 (3 dB point)	70	1.97	31%
59.450	50	1.75	17%
59.470	30	1.60	7%
59.490	10	1.51	0.7%
59.495	5	1.50	~0%
59.499	1	1.50	~0%

presence of the unknown mismatch (see Fig. 5). We may predict the error involved in performing a "long" line measurement for which our criteria, $L \gg cQ_L/f$, is only marginally satisfied or the error in a "short" or "long" line measurement performed off the cavity launcher resonance peak, by returning to (8). Removing the restriction that the measurement be performed at the cavity launcher resonance, $(\omega \neq \omega_o)$, (9) becomes

$$I = \frac{Q_L n_1 E}{\omega_o L\left(1 + jQ_L\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right]\right)}. \tag{14}$$

As the phase is varied, the maximum to minimum signal ratio detected by the crystal coupled to the cavity is now written, (compare with (10))

$$\frac{P_{D\max}}{P_{D\min}} = \left[\frac{Q_{L\max}}{Q_{L\min}}\right]^2 \frac{\left(1 + Q_{L\min}^2\left[\frac{\omega_{\min}}{\omega_o} - \frac{\omega_o}{\omega_{\min}}\right]^2\right)}{\left(1 + Q_{L\max}^2\left[\frac{\omega_{\max}}{\omega_o} - \frac{\omega_o}{\omega_{\max}}\right]^2\right)}. \tag{15}$$

Recalling (6), we see that the VSWR is no longer given by the square root of the ratio of maximum to minimum powers seen by the detector crystal. To estimate the error involved in performing measurements where $\omega \neq \omega_o$, we have tabulated the results given by (15) for a typical case where

$$\begin{aligned}
\frac{Q_{L\max}}{Q_{L\min}} &\equiv \text{VSWR} = 1.5,\\
Q_L &\equiv 450,\\
f_o &\equiv 59.5 \text{ GHz},\\
f_{\max} &\equiv f_o,\\
f_{\min} &\equiv \text{to be varied.}
\end{aligned}$$

These circumstances would occur in a "long" line measurement if one were to try different line lengths in order to vary $\Delta f \equiv f_{\max} - f_{\min}$. To simplify the exercise we have specified that $f_{\max} = f_o$ for all of the different cases so that only $f_{\min}$ is affected by the change in line length. The results of these calculations are shown in Table I.

Examination of the data shown in Table I indicates that less than 1-percent errors are encountered in using (10) in this typical measurement, for $\Delta f \sim 10$ MHz, implying a bandwidth for this technique of a few hundredths of a percent. We note that in general, corrections included in (15) must be estimated for the particular mismatch of interest since $P_{D\max}/P_{D\min}$, in general, depends on Q_L, $\omega_{\min}$, and $\omega_{\max}$, as well as the VSWR of the mismatch.

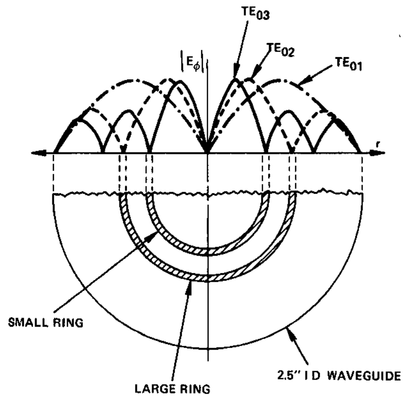


Fig. 7. Relationship of the radial electric-field dependence of the TE_{01}, TE_{02}, and TE_{03} circular waveguide modes to circular rings incorporated in two of the test mismatches.

TABLE II
"LONG" AND "SHORT" LINE VSWR COMPARISON

Type of Mismatch (See Figure 7)	Short Line	Long Line	
	VSWR	VSWR	Δf [Predicted Value = 7.0 MHz]
0.003" Mylar Alone	1.17 ± 0.01	1.23 ± 0.03	6.8 MHz
0.003" Mylar With Small Ring	1.58 ± 0.03	1.44 ± 0.06	6.6 MHz
0.003" Mylar With Large Ring	1.20 ± 0 03	1.24 ± .0.02	6.4 MHz

IV. MODE-SPECIFIC VSWR MEASUREMENTS

A. Comparison: Short Line versus Long Line VSWR Measurements

The VSWR's of various unknown mismatches were analyzed using both the "short" and "long" line measurement procedures. Both methods employed a TE_{02} cavity launcher resonant at 59.5 GHz and a smooth 2° taper section connecting the launcher to 2.5-in ID Cu circular waveguide. Two of the mismatches consisted of 0.003-in mylar sheets, each with a different diameter copper ring attached to the mylar and centered in the waveguide, and a third mismatch was comprised of only the 0.003-in mylar sheet. The relation between the positions of the rings and the field patterns of the TE_{01}, TE_{02}, and TE_{03} modes is shown in Fig. 7. Sections of 2.5-in ID waveguide 20.8 and 382.0-in in length separated the 2° taper and the mismatch for the "short" and "long" line cases, respectively.

Results of these measurements are shown in Table II. "Long" line VSWR results were found by averaging the VSWR results for the four minimum–maximum pairs

TABLE III
MODE DEPENDENT MISMATCH MEASUREMENTS

Mismatch	Launched Mode	VSWR
0 003" Mylar Sheet	TE_{01}	1.19 ± 0.02
	TE_{02}	1.17 ± 0.01
	TE_{03}	1.18 ± 0.04
0.003" Mylar Sheet plus large Cu ring (situated at the TE_{02} radial null)	TE_{01}	1.81 ± 0.04
	TE_{02}	1.20 ± 0.03
	TE_{03}	1.73 ± 0.13
0.003" Mylar Sheet plus small Cu ring (situated at the first TE_{03} radial null)	TE_{01}	1.46 ± 0.03
	TE_{02}	1.58 ± 0.03
	TE_{03}	1.25 ± 0.07

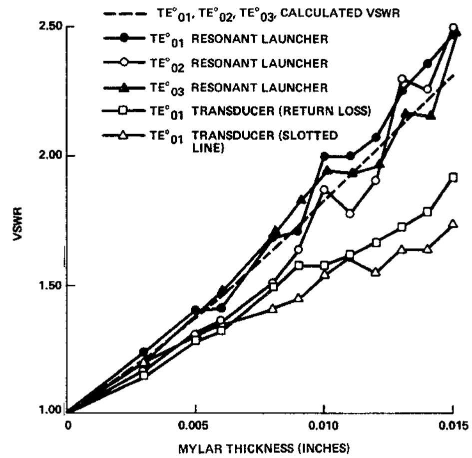


Fig. 8. VSWR of different thicknesses of mylar measured in 2.5-in ID circular waveguide by conventional "return loss" and "slotted line" techniques and the cavity launcher method. Calculated VSWR versus mylar thickness is shown for comparison.

closest to the resonance peak since it was difficult to choose exactly which pair was at resonance (see Fig. 5). The values of VSWR as measured by the two procedures are in rough agreement and estimations of Δf for the long line measurements agree quite well with the predicted value of 7.0 MHz (which includes the dispersive effects of the taper on the phase of the TE_{02} waveguide mode). Both techniques gave low VSWR values for the large ring and high values for the small ring showing that the small ring was located near a radial maximum of the RF electric field and the large ring was located on a radial null of the RF electric field of the TE_{02} mode.

B. Mode-Dependent Mismatches

To emphasize the mode-specific quality of the cavity launcher VSWR measurement technique, we have performed a series of measurements on the mismatches described above for cavities resonant at the same frequency (60 GHz), but in different modes (different sized cavity launchers). Cavity launchers were constructed which launched TE_{01}, TE_{02}, and TE_{03} circular waveguide modes.

Results of these measurements are shown in Table III. As one would expect in examining the relation between the radial dependence of the various modes and the positions of the rings, the small ring mismatch presented a large VSWR to the TE_{01} and TE_{02} modes while the VSWR for the TE_{03} mode was about the same as that for the mylar sheet alone. In a like manner, the large ring mismatch yielded a large VSWR in the presence of TE_{01} and TE_{03} modes,while for the TE_{02} mode, the VSWR was almost identical to that with the mylar sheet alone. This demonstration confirms the ability of the cavity launcher technique to perform mode-specific VSWR measurements.

C. Comparison with Conventional Techniques

As discussed in the introduction, conventional VSWR measurements in multimode waveguide are limited by transducers available between the $TE^{\square}_{10}$ mode and the desired circular waveguide mode. The two most common techniques employing such transducers are the "return loss" and "slotted line" methods shown schematically in Fig. 1. The return loss procedure relies on a measurement of the reflection loss of the mismatch measured using a directional coupler with a given directivity as referenced to a perfect short, while for the "slotted line" method the minimum and maximum amplitudes of the standing wave are measured by probing a slotted rectangular waveguide.

To compare these techniques with the cavity launcher method we have performed a series of VSWR measurements for mismatches consisting of mylar sheets of different thicknesses. The two conventional techniques employed $TE^{\square}_{10} \leftrightarrow TE^{\circ}_{01}$ transducers. All of the measurements were performed at 60 GHz with the mylar sheet placed in 2.5-in ID circular waveguide. Gradual tapers were used to connect either the transducers or cavity launchers for the TE°_{01}, TE°_{02}, and TE°_{03} modes to the 2.5-in waveguide. The length of the taper and waveguide separating the cavity launcher and mismatch was such that only the "short" line method could be used, $L < cQ_L/f$. The results of all these measurements are shown in Fig. 8. In addition, we have plotted the calculated VSWR in 2.5-in ID waveguide for different mylar sheet thicknesses.

First, we observe that the "slotted line" and "return loss" measurement yield VSWR values which are significantly lower than predicted values for thicker mylar sheets. The discrepancy between the conventional VSWR measurements and theory may be due in part to mode conversion in the $TE^{\square}_{10} \leftrightarrow TE^{\circ}_{01}$ transducer. The "slotted line" curves fall below the "return loss" curves because insertion loss in the transducer introduces additional error in the former measurement. Second, we note that the cavity launcher curves coincide quite well with the calculated curve for all mylar thicknesses measured. Not only do these results confirm the accuracy of the cavity launcher

TABLE IV
VSWR FOR MULTIMODE WAVEGUIDE COMPONENTS (TE_{02}^{0} MODE, 59.5 GHz, 2.5" ID WAVEGUIDE)

Component	VSWR
90° Miter Bend	1.06 ± 0.02
Waterload	1.11 ± 0.02
Mode Filter	1.04 ± 0.02
Single-disc Window	1.30 ± 0.02
Double-disc Barrier Window	1.22 ± 0.02

technique, they also highlight the pitfalls in using conventional techniques for multimode VSWR measurements at millimeter wavelengths.

D. Multimode Waveguide Components

Using a 59.5 GHz $-$ TE_{02} cavity launcher, we have measured the VSWR of several multimode waveguide components for use in 2.5-in ID circular waveguide. The results of these measurements are listed in Table IV. Such components are employed in systems handling the high-power microwave output from gyrotron oscillators. We observe that most of the components have a low VSWR, and in the case of the mode filter, the VSWR is low enough that the waveguide gap introduced in using the "short" line technique may affect the accuracy of the measurement as discussed in Section III. All measurements are performed with gaps less than $\lambda/2$ (see Fig. 6) so that errors of the order of 0.02 are present in these measurements. It should be noted that for the single-disc and double-disc barrier windows evaluated here, the VSWR is critically dependent on frequency. In this case, the windows were designed for operation near the cavity launcher resonance, but much higher values of VSWR are measured when the frequency is varied from the design value.

V. CONCLUSION

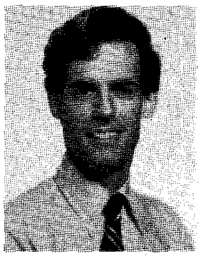
The measurement procedure we have described was developed in order to characterize mismatches used in multimode waveguide systems employing high-power gyrotrons. The technique is limited by the VSWR of the waveguide gap (typically less than 1.05 : 1) when using a short transmission line and the bandwidth of a given cavity launcher is quite narrow ($\sim$ 0.03 percent) for both long and short transmission lines. However, the technique is simple to employ and provides accurate measurements for a wide range of multimode waveguide applications. Ultimately, the technique could be used to characterize the load observed by the gyrotron in a magnetic confinement fusion reactor: the multimode waveguide transmission line leading to the reactor vessel and the thermonuclear plasma itself. Such a measurement, if performed in today's experimental magnetic confinement devices, would provide vital information for the design and operation of gyrotrons for use in heating fusion plasmas.

ACKNOWLEDGMENT

The authors are indebted to R. Symons and M. Chodorow for many helpful discussions.

REFERENCES

[1] H. Jory, S. Evans, K. Felch, J. Shively, and S. Spang, "Gyrotron oscillators for fusion heating," in *Proc. 3rd Joint Grenoble–Varenna Int. Symp., Heating Toroidal Plasmas*, Mar. 1982.
[2] J. F. Shively, and D. S. Stone, "Development of a 200 kW, 60 GHz gyrotron," in *IEDM Tech. Dig.*, pp. 186–187, Dec. 1981.
[3] K. E. Kreischer and R. J. Temkin, "Mode excitation in a gyrotron operating at the fundamental," *Int. J. Infrared and Millimeter Waves*, vol. 2, pp. 175–196, Mar. 1981.
[4] H. G. Effemey, "A survey of methods used to identify microwave fields on wave modes in cylindrical waveguides," IEE paper no. 2837R, pp. 75–83, Jan. 1959.
[5] D. S. Stone, "Mode analysis in multimode waveguides using voltage traveling wave ratios," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 91–95, Feb. 1981.
[6] Marie transducers are manufactured by Marconi Communication Systems Limited, Billercay, Essex, England.
[7] S. E. Miller, "Coupled wave theory and waveguide application," *Bell Syst. Tech. J.*, vol. 33, p. 661, 1954.
[8] C. P. Moeller, "A 60 GHz microwave system design for an ECH experiment on Doublet III," *Bull. Amer. Phys. Soc.*, vol. 26, p. 1036, Sept. 1981.
[9] Robert D. Wengenroth, "A mode transducing antenna," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 332–334, May 1978.
[10] S. Shimada, "Resonant cavity type mode transducer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp. 384–391, Aug. 1966.
[11] H. Derfler, T. J. Grant, and D. S. Stone, "Loaded *Q*'s and field profiles of tapered cylindrical gyrotron cavities," *IEEE Trans. Electron Devices*, vol. ED-29, pp. 1917–1929, Dec. 1982.
[12] N. Morita and Y. Nakanishi, "Circumferential gap in a TE_{01} mode transmitting multimode circular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 183–189, Mar. 1968.
[13] Samuel P. Morgan, "Mode conversion losses in transmission of circular electric waves through slightly non-cylindrical guides," *J. Appl. Phys.*, vol. 21, pp. 329–338, Apr. 1950.



David S. Stone (M'83) was born in Boston, MA in 1952. He received the B.A. degree in physics from Middlebury College, Middlebury, VT, in 1974, and the Ph.D. degree in physics from the Massachusetts Institute of Technology, Cambridge, MA in 1979.

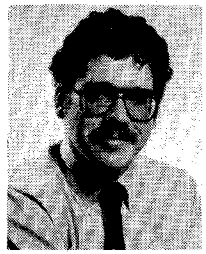
While in graduate school, he specialized in the field of experimental plasma physics at the M.I.T. Research Laboratory of Electronics. His studies covered such topics as tokamak design and engineering, plasma diagnostics and instrumentation, and observation of plasma cyclotron emission. In 1979, he joined the Microwave Tube Division of Varian Associates, Palo Alto, CA as the design engineer in the 60-GHz Gyrotron Development Program, funded by the U.S. Department of Energy. In this capacity he conducted research on the properties of overmoded resonators and multimode transmission lines. This work led to successful testing of a 60-GHz 200-kW pulsed gyrotron in 1981. He then served for one year as Engineering Manager for High Power Klystrons at Varian where he was engaged in production and development engineering for multimegawatt klystrons employed in long-range radars. Currently, he is Engineering Manager for Gyro Amplifiers at Varian. His group is engaged in development of amplifiers based on the cyclotron resonance interaction for use in advanced millimeter radar and communications systems.

Dr. Stone is a member of Phi Beta Kappa, Sigma Xi, and the American Physical Society.

Kevin L. Felch was born in Denver, CO in 1952. After receiving the B.A. degree in physics from Colorado College, Colorado Springs, in 1975, he attended Dartmouth College, Hanover, New Hampshire, where he received the Ph.D. degree in physics in 1980. While at Dartmouth, he carried out experiments observing the microwave Cerenkov radiation produced when a relativistic electron beam is injected into a dielectric-lined lined waveguide, and participated in various free-electron laser studies.

From 1980–1981 he spent a year doing post-doctoral research at the Laboratoire PMI of the Ecole Polytechnique, Palaiseau, France. This work involved a series of free-electron laser studies using intense relativistic electron beams, as well as investigations of the energy disposition of dense electron beams in thin foil targets. In 1981 he joined the Microwave Tube Division of Varian Associates, Inc., Palo Alto, CA, where he is a design engineer in the gyrotron program. This work is aimed at producing high power, high-frequency microwave radiation for plasma heating and communications applications.

Dr. Felch is a member of the Plasma Physics Division of the American Physical Society.



Stephen T. Spang (S'80, M'81) received the B.S.E.E. degree with distinction from Cornell University, Ithaca, N.Y., in May 1981.

While at Cornell, he was involved with the Cooperative Engineering Program. He served as a Design Engineer for bubble memories and as a Product Engineer for CMOS logic circuits at National Semiconductor. He joined the Varian Gyrotron Engineering Group in 1981 and is involved with the testing of 60-GHz oscillators and development of millimeter-wave cold test techniques for cavities, window, and waveguide components.

Mr. Spang is a member of Eta Kappa Nu.

On the Problem of Applying Mode-Matching Techniques in Analyzing Conical Waveguide Discontinuities

GRAEME L. JAMES

Abstract —**Mode-matching techniques in the past have been successfully used to analyze rectangular and circular waveguide problems involving transverse discontinuities. The extension of this method to conical waveguide discontinuities is shown to exhibit difficulties of convergence caused by the behavior of the cutoff conical modes. To illustrate the problem, the junction of a smooth-walled cylindrical waveguide with a corrugated conical horn is discussed in some detail.**

I. Introduction

THE SOLUTION to a transverse discontinuity in a rectangular or circular waveguide using mode-matching techniques has been shown to provide an accurate means of determining the properties created by the discontinuity [1]–[3]. With the properties of the single step established, it is then possible to obtain a solution for any circular or rectangular waveguide which can be considered as a number of transverse discontinuities separated by short lengths of waveguide. This is demonstrated in [4] for the junction between a cylindrical smooth-walled waveguide and a corrugated cylindrical waveguide.

A natural extension of this approach is to analyze transverse discontinuities in conical waveguides. However, in doing so, a number of difficulties arise. This will be demonstrated here by considering the example of the junction between a smooth-walled cylindrical waveguide and a corrugated conical horn. To begin, we review the technique as applied to a small-angle horn where the analysis can be carried out in terms of cylindrical waveguide modes.

II. Junction Between Cylindrical Guide and Small-Angle Corrugated Conical Horn

The radiation pattern of corrugated conical horns are characterized by low sidelobe and low cross-polarization levels. As a result, they are used extensively as feeds in high-performance low-noise reflector antenna systems. To maintain these desirable features, careful design of the throat region of the horn (i.e., the circular-to-conical waveguide junction) is crucial. If the horn semi-angle θ_0 is small

Manuscript received October 19, 1982; revised May 3, 1983.
The author is with the Division of Radiophysics, CSIRO, Sydney, Australia, RPP2660.

0018-9480/83/0900-0718$01.00 ©1983 IEEE